Phenomenology of A Supersymmetric Model for Fermion Mass Hierarchy

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Abstract

Some phenomenological aspects of a supersymmetric model for fermion mass hierarchy proposed previously are discussed. It is required that the lepton universality violation is near to its current experimental bound. The lepton number violation decay modes $\tau \to 2e\mu$ and 3μ maybe observable in the near future. The Majorana mass of electron-neutrino is predicted to be about 0.1 eV. The fine-tuning problem is discussed.

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In this paper, we discuss some phenomenological aspects of a model [1-3] which is proposed for understanding the fermion mass hierarchy within the framework of low energy supersymmetry. In the framework of R-parity violating supersymmetry [4], we thought that, in addition to the Yukawa interactions, the trilinear lepton number violating interactions also contribute masses to the fermions if the sneutrinos have nonvanishing vacuum expectation values (vevs). We further introduced family symmetry among the three generations. The sneutrino vevs make the family symmetry broken. This may give a realistic pattern of fermion mass hierarchy. CP violation occurs superweakly through sneutrino exchanges. Although the model is interesting, it is necessary to discuss its detailed phenomenological implications. We will show that the phenomenological discussions are indeed restrictive to the model. Some predictions will also be made.

Let us first make a review of the model. As for the particle contents, besides all the fields of the minimal supersymmetric extension of the Standard Model, a SU(2)×U(1) singlet superfield X is introduced. In addition to the gauge symmetries which are the same as that in the Standard Model, one discrete family symmetry is introduced. It is a Z_3 symmetry among the SU(2) doublets of the three generations. Denoting L_i and Q_i as the SU(2) doublet superfields of leptons and quarks respectively, with $i = 1, 2, 3$ standing for the three families, this Z_3 symmetry says that the model is invariant under the cyclic operation $(L_1, Q_1) \to (L_2, Q_2) \to (L_3, Q_3) \to (L_1, Q_1)$. Instead of the R-parity, the baryon number conservation is adopted. The superpotential then is

$$
\begin{aligned}
\mathcal{W} \;=\; & g_j^l (\sum_i^3 L_i^a) H_d^b E_j^c \epsilon_{ab} + g_j^u (\sum_i^3 Q_i^a) H_u^b U_j^c \epsilon_{ab} + g_j^d (\sum_i^3 Q_i^a) H_d^b D_j^c \epsilon_{ab} \\
& + \lambda_j (L_1^a L_2^b + L_2^a L_3^b + L_3^a L_1^b) E_j^c \epsilon_{ab} \\
& + \lambda_j^1 (Q_1^a L_1^b + Q_2^a L_2^b + Q_3^a L_3^b) D_j^c \epsilon_{ab} \\
& + \lambda_j^2 (Q_2^a L_1^b + Q_3^a L_2^b + Q_1^a L_3^b) D_j^c \epsilon_{ab} \\
& + \lambda_j^3 (Q_3^a L_1^b + Q_1^a L_2^b + Q_2^a L_3^b) D_j^c \epsilon_{ab} \\
& + \lambda' X (H_u^a H_d^b \epsilon_{ab} - \mu^2) \;,
\end{aligned}
\tag{1}
$$

with a and b being the SU(2) indices. And E^c, U^c and D^c stand for the SU(2) singlet

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antiparticle fields of the leptons, up-type quarks and down-type quarks respectively; H_u and H_d are the two Higgs doublets which couple to the U^c field and D^c field in Yukawa interactions respectively. The term proportional to λ' is nonvanishing so as to avoid a fourth massless neutrino [5] and to break the gauge symmetries. Actually these are the reasons for us to introduce the field X [6].

The soft breaking terms should be added in the Lagrangian. The principle in writing them is gauge invariance and baryon number conservation. We did not assume exact family symmetry in the soft breaking sector. There may be explicitly lepton number violating mass terms $\tilde{\mu}^2 \sum_i A_i^\dagger \phi_d$ with A_i and ϕ_d being the scalar components of L_i and H_d respectively. So that there are no massless majoron as well as light scalar particles [6].

The scalar potential derived in Ref. [1] determines the vevs of the neutral scalar fields. Besides Higgs fields, the scalar neutrinos also get nonzero vevs which are about $10 - 30$ GeV from the previous discussions [2, 3].

Given the model described above, a hierarchical pattern of fermion masses can be obtained [1-3]. The charged lepton and quarks of the third generation get masses from the Yukawa interactions which actually give a kind of democratic mixing of fermions. The muon mass and the down quark mass originate from the tree-level trilinear R-parity violating interactions. Whereas due to the soft breaking of the family symmetry, the electron, the charm quark and the strange quark might obtain masses from loops with neutralino and gluino internal lines respectively. While the pattern of the mass hierarchy arises in an interesting manner in this model, some more careful discussions on the related phenomenology are needed. Some general features of the R-parity violating models have been discussed already [7]. We are going to focus on some specific points of this model.

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1. Lepton universality violation

Without loss of generality, we consider the scenario that only the third sneutrino field has nonvanishing vev which is denoted as v_3. Such a scenario has been discussed elegantly by Ross and Valle [8]. Because $v_3 \neq 0$, the fermions in superfield L_3 mix with that in Higgs and gauge superfields. For simplicity, first we consider the case that the Yukawa and R-parity violating interactions are turned off. For the fermion fields, we found in Ref. [1] that the following composition which are orthogonal to the physical Higgsinos and gauginos are still massless,

$$
\begin{aligned}
\nu_3' &= N_{\nu_3'}\left(\nu_3 - \frac{v_3 v_d}{v_d^2 + v_u^2}\tilde{\phi}_d^0 + \frac{v_3 v_u}{v_d^2 + v_u^2}\tilde{\phi}_u^0\right) , \\
e_3' &= N_{e_3'}\left(e_3 - \frac{v_3}{v_d}\tilde{\phi}_d^-\right) ,
\end{aligned}
\tag{2}
$$

with $N_{\nu_3'}$ and $N_{e_3'}$ being the normalization constants. In Eq.(2), (ν_3, e_3) is the fermionic component of the superfield L_3. $\tilde{\phi}_d$ and $\tilde{\phi}_u$ are the fermionic components of the superfields H_d and H_u which have vevs v_d and v_u respectively. Besides the fermions (ν_1, e_1) and (ν_2, e_2) in the superfields L_1 and L_2, (ν_3', e_3') is of course the third lepton doublet. At this stage, all of the three lepton doublets are weak eigenstates. Now turn on the Yukawa interactions. It can be shown [2] that the charged leptons are still of democratic mixing [9] which results in the mass eigenstate of the left-handed τ,

$$
\tau_L = \frac{1}{\sqrt{3}}(e_1 + e_2 + e_3') .
\tag{3}
$$

At this stage, muon and electron are massless, so that the physical eigenstates of them cannot be uniquely fixed. They can be parameterized as follows,

$$
\begin{aligned}
\mu_L &= \frac{1}{\sqrt{2}}(e_1 - e_2)\cos\theta + \frac{1}{\sqrt{6}}(e_1 + e_2 - 2e_3')\sin\theta , \\
e_L &= -\frac{1}{\sqrt{2}}(e_1 - e_2)\sin\theta + \frac{1}{\sqrt{6}}(e_1 + e_2 - 2e_3')\cos\theta ,
\end{aligned}
\tag{4}
$$

where θ cannot be determined until the muon mass basis is fixed.

After the involvement of R-parity violation, muon gets mass. From Eq. (1) we see that in terms of the mass eigenstates (e_L, μ_L, τ_L), the relevant part for the leptons in

the superpotential is

$$\mathcal{W}' = g^\tau L_\tau^a H_d^b E_\tau^c \epsilon_{ab} + L_e L_\mu (\lambda_\mu E_\mu^c + \lambda_\tau E_\tau^c) \ . \tag{5}$$

E_τ^c is propotional to $g_j^l E_j^c$. g^τ and λ_μ, λ_τ are the Yukawa coupling constant and the R-parity violating coupling constants in the mass eigenstates, respectively. They are compositions of g_j^l, λ_j ($j = 1,\ 2,\ 3$) in Eq. (1). E_μ^c is a combination of E_j^c, which is orthogonal to E_τ^c. It will be proved to be the right-handed muon mass eigenstate. Up to now, it is not neccessarily the mass eigenstate, however we always have the freedom to write the superpotential in the above form. Assumption that only ν_3 gets a vev leads that $\cos\theta = 1$, and it is at this stage we see E_μ^c is just (approximately) corresponding to the mass eigenstate of the right-handed muon.

It is because ν_3' and e_3' in Eq. (2) do not coincide in form that makes the leptons deviate from their universality. Compared to the $e_1 \rightarrow \nu_1$ weak transition, the $e_3' \rightarrow \nu_3'$ transition amplitude is suppressed by a factor of $C = N_{\nu_3'} N_{e_3'}(1 + \frac{v_3^2}{v_d^2 + v_u^2})$. For $e \rightarrow \nu_e$ weak transition, where $\nu_e \equiv \frac{1}{\sqrt{6}}(\nu_1 + \nu_2 - 2\nu_3')$, the suppression factor is $\tilde{C}_e = \frac{1}{3}(1 + 2C)$ which can be effectively absorbed into the gauge interaction coupling constant g^e. For $\mu \rightarrow \nu_\mu$ weak transition, due to $(\nu_3',\ e_3')$ is not the composition, the corresponding factor $\tilde{C}_\mu = 1$. For the $\tau \rightarrow \nu_\tau$ weak decay, where $\nu_\tau = \frac{1}{\sqrt{3}}(\nu_1 + \nu_2 + \nu_3')$, the suppression factor is $\tilde{C}_\tau = \frac{1}{3}(2 + C)$ which can be effectively absorbed into the gauge interaction coupling constant g^τ. Therefore $\tilde{C}_l \neq 1$ ($l = e,\ \tau$) just measures the lepton universality violation. With reasonable choice of $\tan\beta$, like $\tan\beta \simeq 2.2$ [3], if v_3 were taken as 30 GeV, the violation of factor C from unity would be as large as 10%. Only if $v_3 \leq 10$ GeV, can this violation be within 0.4%, and hence the violation of the factors $\tilde{C}_l$ from unity smaller enough to satisfy the current experiment [10]. However, the value of v_3 cannot be too small in this model, because it contributes muon mass. We will take v_3 as 10 GeV in the following. With such choice of parameters, we have the following

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lepton universality violation,

$$
\begin{aligned}
g^e : g^\mu : g^\tau &= \tilde{C}_e : \tilde{C}_\mu : \tilde{C}_\tau = \tfrac{1}{3}(1+2C) : 1 : \tfrac{1}{3}(2+C) \\
&= 0.997 : 1 : 0.999 = 1 : 1.003 : 1.002 \ ,
\end{aligned}
\tag{6}
$$

which is still consistent with experiments [10].

It should be remarked here that the above discussion on the lepton universality violation is different from that in Refs. [5] and [8]. We have introduced the coupling λ' in this model (see Eq. (1)), there is no fourth massless neutrino. So the violation cannot be rotated away by redefining the τ-neutrino field.

2. Lepton number violation and FCNC

Generally this model allows for lepton number violation processes at tree level. The violation is caused by the trilinear R-parity violating terms with the slepton exchange. From Eq. (5), it is interesting to see that the decay $\mu \to 3e$ does not occur in this model. Therefore, this model avoids one of the most stringent restriction of the lepton number violation. Lepton number violation occurs in τ rare decays. If we consider the processes with only charged leptons as final states, the decay modes $\tau \to 2e\mu$ and $\tau \to 3\mu$ can occur, as shown explicitly in Figure 1. They have the same decay rate,

$$
\Gamma = \frac{[\frac{\lambda_\mu \lambda_\tau}{m_{\tilde{\nu}}^2}]^2 m_\tau^5}{192\pi^3} \ .
\tag{7}
$$

From the muon mass $m_\mu \simeq \lambda_\mu v_3$, we get $\lambda_\mu \sim 10^{-2}$. It is reasonable to assume that the value of λ_τ has the same order of magnitude as λ_μ. In this case, the branching ratio of the decays $\tau \to 2e\mu$ and 3μ are $10^{-7} - 10^{-11}$ if the sneutrino mass is around $100 - 1000$ GeV. Such a branching ratio maybe observable in the near future. The modes $\tau \to 3e$ or $2\mu e$, however cannot occur in this model.

A remark should be made on the slepton SUSY breaking parameters. The lepton flavor violation, like in the proccess $\mu \to e\gamma$, can be induced by the SUSY breaking

parameters through loops. The experimental data on such process put severe constraints on these parameters. The symmetries assumed in this model, however, do not constrain these parameters strong enough. Similar to what have been done for the quark case [3], further assumptions are needed here. They are (i) the slepton mass-squared matrice corresponding to definite chiralities of leptons are proportional to unit matrix; (ii) the slepton mass-squared matrix corresponding to the mixing associated with different chiralities is propotional to the lepton mass matrix.

However, these conditions cast doubt on the electron and neutrino mass generation mechanism through loops proposed by Ref. [2]. In that mechanism, we hoped to violate condition (i) to give electron mass through loop. But the violation is required to be small and can be measured by some dimensionless quantities δ, that are defined as square of the ratios of the flavor off-diagonal masses to the average slepton mass. From the experimental constraints [11], $\delta \leq (10^{-2} - 10^{-3}) \times (\frac{m_{\tilde{l}(\text{GeV})}}{100})^2$, when the averaged slepton mass $m_{\tilde{l}}$ is compatible with the photino mass $m_{\tilde{\gamma}}$. According to Ref. [2], the electron mass should be $m_e \simeq \frac{\alpha}{4\pi}\delta\frac{\tilde{m}v_d}{m_{\tilde{l}}}$, where $\tilde{m}$ is soft mass parameter associated with different chirality mixing, which might give realistic numerical result of electron mass, if $\tilde{m} \simeq 1$ TeV were taken. Such a value of $\tilde{m}$ is too large to be acceptable.

3. Neutrino masses

The loop induced Majorana neutrino mass is inevitable in this model. The best way in studying this problem is to work in the basis of the mass eigenstate. The mechanism for the loop-level neutrino mass was discussed in Ref. [12]. From Eq. (5), we see that only electron type neutrino can be massive through loop as shown in Figure 2, the induced mass is

$$
\begin{aligned}
m_{\nu_e} &\simeq \frac{(\lambda_\mu)^2}{16\pi^2}\frac{\tilde{m}^0 m_\mu^2}{m_{\tilde{l}}^2}, \\
&\simeq \frac{(\lambda_\mu)^2}{16\pi^2}\frac{\tilde{m}v_d}{m_{\tilde{l}}^2}\frac{m_\mu^2}{m_\tau} .
\end{aligned}
\tag{8}
$$

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In above equation, we reexpressed the parameter $\tilde{m}$ by $\tilde{m}^0$ which is a more natural choice in the lepton case. For reasonable value of $\tilde{m}^0$ and $m_{\tilde{l}}$, $\tilde{m}^0 \simeq m_{\tilde{l}} \simeq 100$ GeV, we predict Majorana ν_e mass is around 0.1 eV. Note that only muon and its superpartner contribute the ν_e mass in the loop. At this stage, the model predicts vanishing masses of ν_μ and ν_τ. And there is no mixing between ν_e and them.

It is necessary to discuss the tree-level induced neutrino mass. Usually in the case of large sneutrino vev, we expect that, because of the mixing with Zino, neutrino gets a large mass by see-saw mechanism unless an unnatural fine-tuning is made. But this model avoids such tree-level neutrino mass, as shown explicitly in the following. The Lagrangian for the neutralino masses is given as

$$
-i(\nu_3 \;\; \tilde{\phi}_d^0 \;\; \tilde{\phi}_u^0 \;\; \tilde{Z} \;\; \tilde{X})
\begin{pmatrix}
0 & 0 & 0 & av_3 & 0 \\
0 & 0 & 0 & av_d & \lambda'v_u \\
0 & 0 & 0 & -av_u & \lambda'v_d \\
av_3 & av_d & -av_u & M_{\tilde{Z}} & 0 \\
0 & \lambda'v_u & \lambda'v_d & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\nu_3 \\
\tilde{\phi}_d^0 \\
\tilde{\phi}_u^0 \\
\tilde{Z} \\
\tilde{X}
\end{pmatrix}
+ \text{h.c.,} \quad (9)
$$

where $a = (\frac{g^2+g'^2}{2})^{1/2}$ with g and g' the SU(2)×U(1) gauge coupling constants. Note that the Zino mass $M_{\tilde{Z}}$ of the soft breaking term has also been included in. It is easy to see that the above mass matrix is of rank 4 (instead of 5) with eigenstate ν_3' expressed in Eq. (2) corresponding to the zero eigenvalue. However, this does not mean that there is no fine-tuning problem in this model. This problem has actually transferred to the smallness of the coupling constants of the terms like $H_u H_d$, $H_u L_i$ and $X H_u L_i$ which we have not written in Eq. (1). While the smallness needs further explanation, less fine-tuning is needed in this model because of supersymmetry.

In summary, the supersymmetric model for fermion mass hierarchy proposed in Refs. [1-3] has got restrictive limitations from phenomenological considerations. It requires that the lepton universality violation is near to its experimental bound. The

lepton number violation decay modes $\tau \to 2e\mu$ and 3μ maybe observable in the future's experiments. The electron neutrino Majorana mass is predicted to be about 0.1 eV.

Finally a remark should be made on the electron mass. This model gives an interesting hierarchical pattern for leptons. However the idea of radiative generation for electron mass [2] is seriously problematic. Actually such kind of idea does not make simplification torwards the understanding of the lepton mass hierarchy, instead it merely transfers the problem from the Yukawa sector to other sector of the model. Therefore it is fair to say that the idea of radiative mass generation is not appealing in the framework of this model. It is preferable if the electron mass originates in some new physics which has not yet been included in the Lagrangian.

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FIGURE CAPTIONS

Fig. 1   Feynman diagrams for the decays $\tau \to 2e\mu$ and 3μ.

Fig. 2   The mechanism for electron-neutrino mass generation.
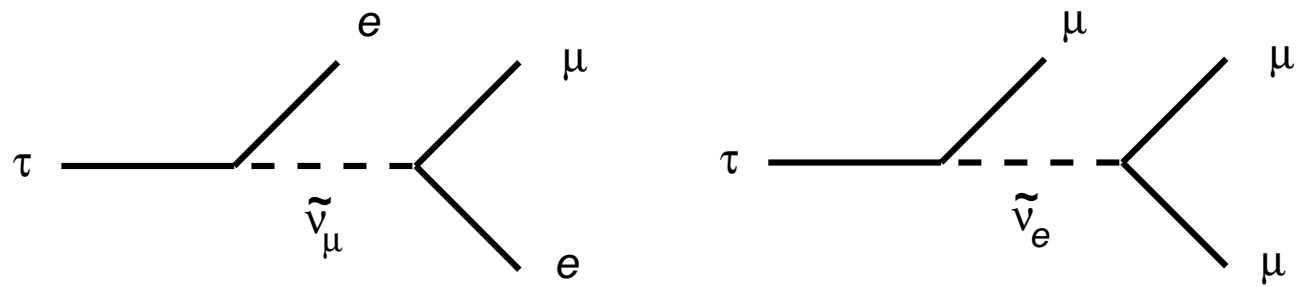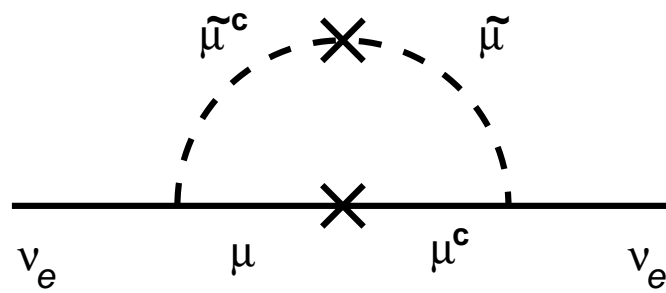
Fig. 1

Fig. 2

FIGURES